International Journal of Science and Engineering Applications
Volume 6–Issue 12, 372-381, 2017, ISSN:-2319–7560

Efficient Chromatic and Residual Dispersion Postcompensation for Coherent Optical OFDM

Ishiwu I. Jude
Department of Electrical/Electronics Engineering
Modibbo Adama University of Technology, Yola
Adamawa State, Nigeria

Yahya Adamu
Department of Electrical/Electronics Engineering
Modibbo Adama University of Technology, Yola
Adamawa State, Nigeria

Oguche D. Onoja
Department of Electrical/Electronics Engineering
Modibbo Adama University of Technology, Yola
Adamawa State, Nigeria

Boyson Andrew
Department of Electrical/Electronics Engineering
Modibbo Adama University of Technology, Yola
Adamawa State, Nigeria

Abstract: In lieu of other impairments associated with fiber communication such as; fiber nonlinearity, fading, Intersymbol Interference (ISI), Intercarrier Interference (ICI), Chromatic Dispersion (CD) is compensated by Coherent Optical Orthogonal Frequency Division Multiplexing (CO-OFDM) technique. This technique divides the available bandwidth into five subbands, each modulated at a low data rate and postcompensated for the Chromatic Dispersion. Implementation of the Optical OFDM involves the use of Digital Signal Processing (DSP); Inverse Discrete Fourier Transform (IDFT) and Discrete Fourier Transform (DFT) both at the transmitter and receiver respectively. The Residual Dispersion left after CD is compensated by Constellation Adjustment Method (CAM). Simulation results using Optisystem show 107-Gb/s single-channel transmission over 1000-km Standard Single Mode Fiber (SSMF) with polarization division multiplexing Four Quadrature Amplitude Multiplexing (4-QAM) using 128 DFT, 82 subcarriers, 5 pilot subcarriers and 16 guard intervals. Equally, the simulation analysis were done at various transmission distances, OFDM systems show a better Min. BER and Max. Q factor than the conventional Non-Return to Zero (NRZ) systems.

Keywords: BER, Chromatic Dispersion, CO-OFDM, Cyclic Prefix, DFT, Residual Dispersion.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier transmission technique, which divides the available spectrum into many carriers, each one being modulated by a low rate data stream. OFDM is similar to Frequency Division Multiplexing Access (FDMA) in that the multiple user access is achieved by subdividing the available bandwidth into multiple channels, which are then allocated to users [1]. However, OFDM uses the spectrum much more efficiently by spacing the channels much closer together. This is achieved by making all the carriers orthogonal to one another, preventing interference between the closely spaced carriers.

OFDM is a widely used and very attractive modulation and multiplexing technique for broadband wireless and wired communication system due to its spectrum efficiency and channel robustness . OFDM belongs to a broader class of Multi-Carrier Modulation (MCM) carrying data over many lower rate subcarriers[2]. OFDM, forms the basis of many of several telecommunications standards in the world, counting from Digital Terrestrial Television (DTT), Wireless Local Area Networks (LANs), digital radio broadcasting and 4G mobile communications [3]. OFDM is also the source of nearly all Digital Subscribers Line (DSL) standards, and within this context, OFDM is generally known as Discrete Multi-Tone (DMT) because of some minor peculiarities.

Though, OFDM offer tremendous benefits and its wide-spread use in wireless communications, it has been considered for optical communications during the last years [4]. The need to develop a high-speed and robust communication services and the tremendous expansion of the Internet are driving the development of high-capacity and flexible optical transport networks. In recent times, many researches started to pay more interest to apply the OFDM technique with MCM, instead of Single Carrier Modulation (SCM) in optical fiber communication due to its ability to reduce the effect of selective fading, chromatic dispersion, Inter-Symbol Interference (ISI) and Inter-Carrier Interference (ICI).

Optical OFDM has gained much interest in recent years as it is developed for long-haul transmission network or rather longer distance transmission and has capability to equalize Chromatic Dispersion (CD) and Polarization Mode Dispersion (PMD) efficiently. OFDM technique has been applied so it can be utilized in Wavelength Division Multiplexing (WDM) system [5]. To attain a high spectral efficiency and achieve simple channel equalization, OFDM takes benefit of the Fast Fourier Transform (FFT) [6, 7]. Optical OFDM has become one of the most capable technologies that are used for designing bit rate and bandwidth variable transponders for spectral efficient optical networks. O-OFDM with phase modulation and coherent detection is also the future for suitable spectral efficient key, robust against system nonlinearities, and for transmission in elastic networks[8, 9].

2. THEORY

2.1 Historical Background of OFDM

Chang in 1966, first introduced the concept of OFDM in a seminal paper, he proposed a method to synthesize band limited signals for multi channel transmission. The idea is to transmit signals simultaneously through a linear band limited channel without Inter Carrier Interference (ICI) and Inter Symbol Interference (ISI) [1]. Based on Chang`s work, Salzberg in 1967, performed the analysis and came up with conclusion that the focus to design a multi channel transmission must concentrate on reducing crosstalk between adjacent channels rather than on perfecting the individual signals. In fact, "OFDM" first appeared in his separate patent in 1970 [10].

The proposal to create the orthogonal signals using an FFT came in 1969 [11]. In 1980, the cyclic prefix (CP) was proposed [12]. For practical wireless applications, OFDM began to be considered in the mid 1980s. A paper on OFDM for mobile communications is published by Cimini of Bell Labs in 1985 [3]. In 1987, the use of OFDM for radio broadcasting is considered and the significance of combining Forward Error Correction (FEC) with OFDM is also noted by Lassalle and Alard [13]. Table 1 shows some of the key milestones of the OFDM technique in radio frequency (RF) domain.

Table 1. Historical Development of RF OFDM

S/No	Date	Author(s)	Contributions
1	1966	R. Chang	Foundation work on OFDM
2	1967	Salzberg	Performed the analysis based on Chang`s work
3	1969	S.B. Weinstein and P.M Ebert	FFT implementation of OFDM
4	1980	R.Peled and A. Ruiz	Cyclic prefix was proposed
5	1985	L. Cimini	OFDM for mobile communications
6	1987	Alard	Combination of FEC and OFDM
7	1995	ETSI	DSL formally adopted DMT, a variation of OFDM
8	1995 (1997)	ETSI	European Telecommunications Standards Institute (ETSI) Digital Audio (Video) Broadcasting standard, DAB(DVB)
9	1999 (2002)	IEEE Standard	Wireless LAN standard, 802.11 a (g), Wi-Fi
10	2004 (2007)	IEEE WiMax Forum	Wireless MAN standard, 802.16, WiMax
11	2009 (2015)	3GPP/ESTI	Long Term Evolution (LTE), 4 G mobile standard

Although, OFDM has been studied in RF domain for over four to five decades, the research on OFDM in optical communication began only in the late 1990s by the two pioneers; Pan and Green. In the late 2000s, long-haul transmission by optical OFDM has been investigated by a few groups. Two major research directions appeared, Direct Detection Optical OFDM (DDO-OFDM) looking into a simple realization based on low-cost optical components and Coherent Optical OFDM (CO-OFDM) aimed to achieve high spectral efficiency and receiver sensitivity, though with inherent complex circuit design due to digital signal processing. Since then, the interest in optical OFDM has increased dramatically. In 2007, the world's first CO-OFDM experiment with line rate of 8-Gb/s was reported [14]. In the last few years, the transmission capacity continued to grow tremendously about ten times per year. In 2009, up to 1 Tb/s optical OFDM was successfully demonstrated [15]. Table 1 shows the progress of optical OFDM in the last two decades.

Table 2. Progress of Optical OFDM

S/No	Date	Author(s)	Contributions
1	1996	Pan and Green	OFDM for Community Access Television
2	2001 2001	You and Kahn Dixon et al.,	OFDM in DD System OFDM over Multimode Fiber
3	2005 2005	Jolley et al., Lowery and Armstrong	Experiment of 10Gb/s O-OFDM over Multimode Fiber Power-efficient O-OFDM in DD Systems
4	2008	Djordjevic and Vasic Shieh and Athaudage	Long-haul transmission DDO-OFDM Long-haul transmission CO-OFDM
5	2010	Shieh et al.,	8-Gb/s CO-OFDM transmission over 1,000 km
6	2014	Yang et al	56Gb/s and 1100-Gb/s per single channel CO-OFDM using OBM
7	2016	Ma et al.,	8x520-Gb/s Signal Based on Signal Band/λ PDM-16QAM on 75-GHz Grid.
8	2017	Dar et al.,	Chromatic dispersion compensation techniques and characterization of fiber Bragg grating for dispersion compensation

Besides offline DSP, from 2009 onward, a few research groups started to investigate real-time optical OFDM transmission. The first real-time optical OFDM demonstration took place in 2009; three years later, then real-time single-carrier coherent optical reception was demonstrated. The pace of real-time OFDM development is fast, with the net rate crossing 10-Gb/s within one year. Moreover, by using Orthogonal-Band-Multiplexing (OBM), which is a key advantage for OFDM, up to 56-Gb/s and 110-Gb/s over 600-km Standard Single Mode Fibre (SSMF) was successfully demonstrated. Recently, in 2016 another breakthrough was made, the transmission of 8x520-Gb/s Signal Based on Signal Band/λ Pulse Density Modulation (PDM)-16QAM-OFDM on a 75-GHz Grid [16]. As evidenced by the commercialization of single-carrier coherent optical receivers, it is foreseeable that real-time optical OFDM transmission with much higher net rate will materialize in the near future based on state-of-the-art Application Scientific Integrated Circuits (ASIC) design.

2.2 Optical Fibre Communications

The communication system of fibre optics is well understood by studying the parts and sections of it. The major elements of an optical fibre communication system are shown in Figure 1.

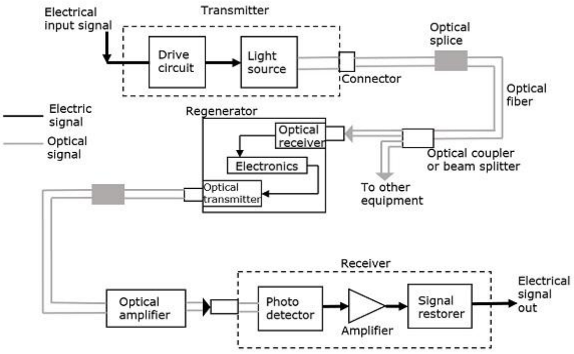


Figure 1. Block Diagram of Optical Fibre Communication System.

The basic components are light signal transmitter, the optical fibre, and the photo detecting receiver. The additional elements such as fibre and cable splicers and connectors, regenerators, beam splitters, and optical amplifiers are employed to improve the performance of the communication system.

2.3 Principle of orthogonality

Orthogonality is a property which allows multiple signals to be perfectly transmitted over a common channel and detected at the receiver without interference. Below are the three cases where frequency orthogonolality is achieved.

CASE 1: From Figure 2 below, the symbol of each OFDM has a duration, $T_s = N/R$. Expressing the OFDM signal in the time domain $S_k(t)$ as the summation of each information symbol X_{ik} being carried in the kth subcarrier within the ith OFDM symbol. On the choice of modulation used for the subcarriers, complex values are resulted by superposition of the subcarriers, although not accounted yet on this research. The period of the OFDM symbol becomes;

$$S_k(t) = R\{ \sum_{i=-\infty}^{\infty} \sum_{k=0}^{N-1} X_{i,k} e^{j2\pi fkt}. P(t-iTs)\} \qquad 1$$

Where; $P(t)$ represent an ideal square pulse of length,

N, represent the number of subcarrier

while; F_k, represent the subcarrier frequency.

In equation 1, the OFDM symbol is ideally multiplied by a square pulse $P(t)$, which is one for a T_s-second period and zero otherwise. The amplitude spectrum of that square pulse has a form $\sin(c\pi f t)$, which has zeros for all frequencies f that are an integral multiple of $1/T_s$.

CASE 2: Consider a subcarrier frequency of the equation below;

$$F_k = K\frac{1}{Ts} \qquad 2$$

We can deduce from the above equation that each subcarrier must be separated from its neighbours by exactly $1/T_s$, so each subcarrier within an OFDM symbol has exactly an integer number of cycles in the interval T_s, and the number of cycles differs by exactly one, as depicted in Figure 2. This way, orthogonality between subcarriers is achieved.

CASE 3: Consider another scenario where orthogonality is achieved, this property can be explained for any couple of subcarriers by the following expression:

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} \cos\left(\frac{2\pi mt}{T}\right) \cos\left(\frac{2\pi nt}{T}\right) dt=0,\ m\neq0 \qquad 3$$

We consider a situation where m & n are different natural numbers, the area under this product over one period is zero as this can be shown in Figure 2.9 for a three subcarriers. The frequencies of these waves are called harmonics and for these, the orthogonality condition is always fulfilled.

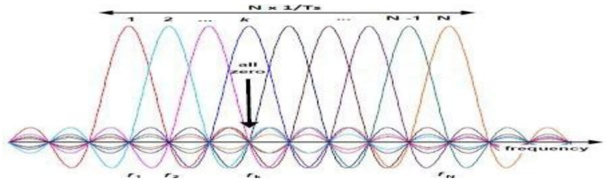


Figure 2. Time Domain Subcarriers within an OFDM symbol

2.4 Chromatic Dispersion (CP)and Equalization

Chromatic dispersion is a deterministic distortion given by the design of the optical fibre. It leads to a frequency dependence on the rate at which the phase of the wave propagates in space (optical phase velocity) and its effect on the transmitted optical signal basically scales quadratically with the data rate [17].

This frequency dependence of the phase can be easily identified by describing a pulse propagating through a monomode optical fibre in the frequency domain:

$$X_{out}(\omega) = X_{in}(\omega).e^{j\beta(\omega)z} \qquad 4$$

Where $X_{in}(\omega)$ represents the Fourier transform of the transmitted signal, $X_{out}(\omega)$ is the Fourier transform of the received signal and $ß(\omega)$ corresponds to the phase constant of the fundamental propagating mode.

Because of the frequency dependence on ß, the main limiting effect considered in equation 4 will be chromatic dispersion. Other phenomena such as losses or nonlinearities will be not considered, though their effects in fibre propagation can be added afterwards. The consideration of dispersion as the main limiting effect in an optical transmission has been shown to be a good approach in a broad variety of practical applications, but more importantly allows the simplification of its study.

In an ideal case, the phase constant in equation 4 has a linear dependency with frequency, meaning that all the spectral components undergo the same phase delay, which is the same as saying that they travel at the same velocity. At reception the same signal will be obtained without any distortion but with a constant delay. On the other hand, in a dispersive channel the phase constant $e^{j\beta(\omega)z}$ in equation 4, has a nonlinear dependency with frequency and as a consequence of the different arrival times of the frequency components, the recovered signal at the reception end will differ from the transmitted one

In order to obtain an OFDM signal without errors at the receiver, the use of cyclic prefix is essential. This will eliminate ISI when a temporal dispersion affects the channel. However, the effect of chromatic dispersion causes the information symbols to still be affected by amplitude and phase changes when arriving to the receiver, as shown in Figure 3 below.

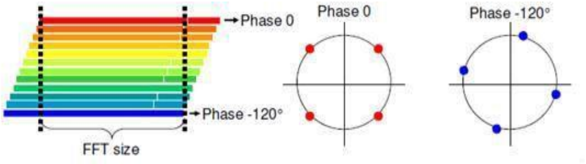


Figure 3. Phase Distortions on the Received Constellation.

Consequently, an N-level equalizing stage has to be introduced right after the FFT operation at the receiver in order to correct the phase and amplitude levels, where N is the number of received subcarriers. The design parameters for this stage should be obtained through a channel estimation, which is usually performed with training sequences. These sequences are added by using pilot subcarriers in each OFDM symbol, so the channel transfer function can be approximated. As the design of training sequences is beyond the scope of this work, the required phase compensation for each OFDM symbol will be calculated based on the dispersion model suffered by each subcarrier:

$$\varphi = \frac{1}{2} B_2 W^2 L \qquad 5$$

Where L is the fibre length, W is the subcarrier frequency, and B is the second term order of the signal phase delay approximation.

However, this equalization will be not enough to obtain the ideal received constellation, as a constant phase shift will still affect the received symbols due to the choice of the reference frequency for the fibre. Hence, the Constellation Adjustment Method proposed in this thesis will eliminate the residual chromatic dispersion left after equalization.

2.5 Coherent Optical OFDM Systems (CO-OFDM) Systems

CO-OFDM represents the ultimate performance in receiver sensitivity, spectral efficiency, and robustness against polarization dispersion, but yet requires the highest complexity in transceiver design. In the open literature, CO-OFDM was first proposed by Shieh et al., and equally formalized the concept of the coherent optical MIMO-OFDM. They carried out an early CO-OFDM experiments over 1,000 km SSMF transmission at 8-Gb/s [14]. Kaneda et al., proposed a field demonstration of 100-Gb/s real-time coherent optical OFDM detection [8].

The conceptual diagram of CO-OFDM system is shown in Figure 4 [18]. It contains five essential functional blocks: 1) RF-OFDM signal transmitter, 2) RF to optical (RTO) up-converter, 3) Optical fibre links, 4) optical to RF (OTR) down-converter, 5) RF-OFDM receiver. Such setup can be also used for single-carrier scheme, in which the DSP part in the transmitter and receiver needs to be modified, while all the other hardware setup remains the same.

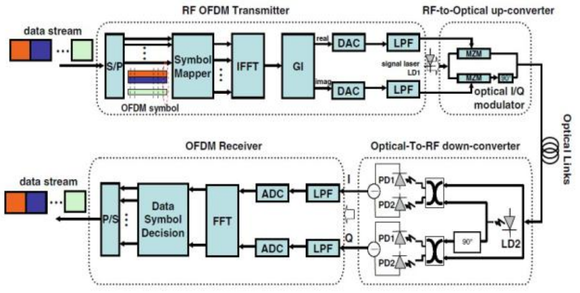


Figure 4. Block diagram of a Coherent Optical OFDM System.

When the modulation technique of OFDM combines with coherent detection, the benefit brought by these two powerful techniques are multifold: (1) High spectral efficiency; (2) Robust to chromatic dispersion and polarization-mode dispersion; (3) High receiver sensitivity; (4) Dispersion Compensation Modules (DCM)-free operation; (5) Less DSP complexity; (6) Less oversampling factor; (7) More flexibility in spectral shaping and matched filtering.

3. System Setup

The optical transmission link with and without equalizer compensation by using single channel CO-OFDM system is setup using an OptiSystem as a simulation tool. It has been used by many researchers to simulate the fibre nonlinearity and dispersion effects in optical communication systems. Simulation setting takes most key optical communication system/component parameters into account including fibre nonlinearity, noise, dispersion, etc. A generic CO-OFDM system includes five basic functional blocks as already shown and fully explained in Figure 4: OFDM transmitter, RF to Optical (RTO) up-converter, optical link, Optical to RF (OTR) down-converter, and OFDM receiver. The general parameters used for this simulation are shown in Table 3 below, while Table 4 and 5 show the Transmitter/Receiver Parameters, Optical Fibre Link Parameters and Receiver Parameters respectively;

Table 3. Simulation Parameters

S/No	Parameters	Values	Units
1	Bit rate	10e9	bit/sec
2	Time window	12.8e-9	Sec
3	Sample rate	640e9	Hz
4	Sequence length	128	Bits
5	Samples per bit	64	
6	Symbol rate	10e9	Symbols/sec
7	Sensitivity	-100	dB
8	Resolution	0.1	Nm
9	Interpolation offset	0.5	Nm
10	Number of leading zeros`	3	

Table 4. Transmitter/Receiver Parameters

S/No	Parameter	Value	Units
1	QAM Bits Per Symbol (b/sym)	2	bits
2	QAM Gray Code	No	
3	QAM Differential Coding	No	
4	OFDM Size	144	
5	OFDM Maximum possible subcarriers	128	
6	OFDM Guard interval	16	
7	OFDM used subcarrier	82	
9	OFDM unused subcarrier	41	
10	OFDM Pilot subcarrier	5	
11	Average OFDM power	12	dBm
13	CW Laser frequency	193.1	THz
14	CW Laser power	10	dBm
15	CW Laser linewidth	0.1	MHz
16	CW Laser Azimuth	45	degree
17	LPBF Cut-off frequency	0.75 x Symbol rate	Hz

Table 5. Optical Fibre Link Parameters

S/No	Parameter	Value	units
1	Optical fibre length	100 to 1000	km
2	Attenuation	0.2	dB/km
3	Dispersion	16.75	Ps/nm/km
4	Effective area	80	μm^2
5	Reference wavelength	1550	nm
6	Optical gain	20	dB
7	Optical noise figure	4	dB
8	Ideal Dispersion Compensation FBG Bandwidth	125	GHz
9	Dispersion	-800	Ps/nm

3.1 Coherent Optical-OFDM Transmitter

At the transmitter side, both modulation and multiplexing are achieved digitally using an Inverse Fast Fourier Transform (IFFT). The subcarrier frequencies are mathematically orthogonal over one OFDM symbol period. Figure 5 shows the simulation set-up for the 4-QAM CO-OFDM Transmitter. The 107-Gb/s CO-OFDM signal is generated by multiplexing five OFDM sub-bands. In each band, 21.4-Gb/s OFDM signals are transmitted in both polarizations. The QAM CO-OFDM transmitter has a gain and bias of 0.021 volt and 1.5 volt respectively. The optical frequency source is a Continuous Wave (CW) laser of 1550 nm and 10 dBm optical frequency and power respectively. The transmitted signal is generated off-line by a Pseudo-Random Bit Sequence (PRBS) Generator with $10e^9$ bits/secs and mapped to 4-QAM constellation. The digital time domain signal is formed after IFFT operation. The FFT size of OFDM is 128 carriers, and guard interval is 1/8 of the symbol window. The middle 82 subcarriers out of 128 are filled, from which five pilot subcarriers are used for phase estimation.

A Tektronix Arbitrary Waveform Generator (AWG) is used to generate analogue signals for both optical I and Q parts. The optical I/Q modulator comprising two MZMs with 90^0 phase shift is used to directly impress the baseband OFDM signal. The modulator is biased at null point to suppress the optical carrier completely and perform linear baseband-to-optical up-conversion. The signal is then propagated through the optical link and becomes degraded due to fibre impairments.

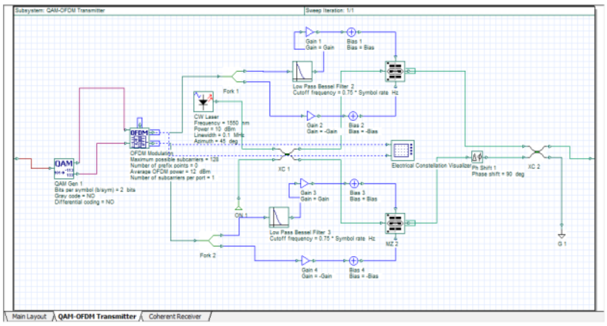


Figure 5. Architecture of 4-QAM CO-OFDM Transmitter Subsystem

3.2 Optical Fibre Link

The Fibre Link for CO-OFDM set-up consists of 100 km span of Standard Single Mode Fibre (S-SMF) including an Ideal Dispersion Compensation Fibre Bragg Grating and Optical Amplifier prior to each span. Figure 6 shows the configuration of the fibre link. The Optical Amplifier increases the link distance, which is limited by fibre loss in an optical communication system. The length of fibre can be increased by increasing the number of loops. The amplifiers have a 4-dB noise figure and a 20-dB gain, while the Optical Fibre has a dispersion of 16.75 ps/nm/km, an effective area of 80 µm^2, attenuation of 0.2-dB/km and reference wavelength of 1550 nm. The ideal dispersion compensation Fibre Bragg Gratings (FBGs) have a frequency of 193.1 THz, bandwidth of 125 GHz, dispersion of -800 ps/nm and a noise threshold of about -100 dB.

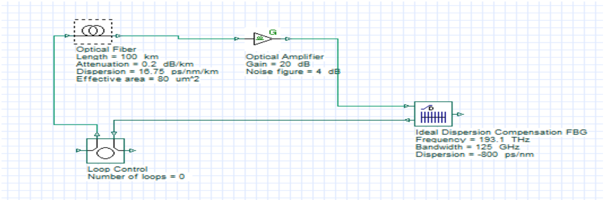


Figure 6. Architecture of CO-OFDM Fibre Link

3.3 Coherent Optical-OFDM Receiver

At the receiver side, a coherent optical receiver comprising a polarization beam splitter, a local laser, two optical 90^0 hybrids, and four balanced photo-receivers is used to down-convert the data to the RF domain. Figure 7 shows the schematic of Coherent Optical-OFDM receiver.

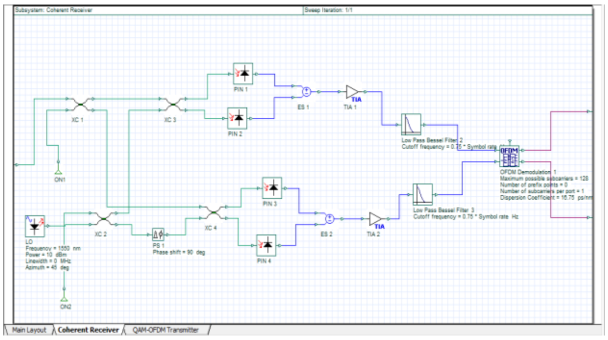


Figure 7. Architecture of Coherent Optical-OFDM Receiver Subsystem.

Finally, the complete architecture of the proposed 107-Gbps 4-QAM CO-OFDM Dispersion Compensation Post Compensated System is shown in Figure 8. However, the input data for the OFDM modulator can have different modulation formats such as BPSK, QPSK, QAM, etc.

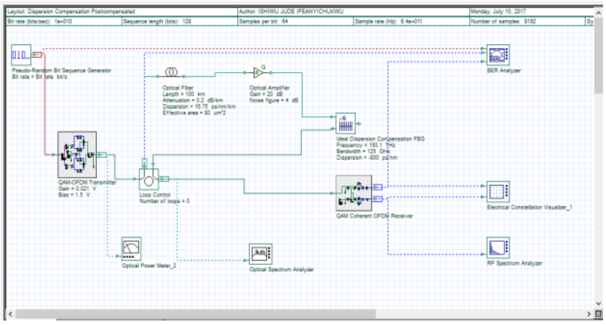


Figure 8. 107-Gb/s 4-QAM CO-OFDM Dispersion Compensation Post Compensated System.

3.4 The Logical Model

The addition of Cyclic Prefix (CP) is done immediately after Digital Signal Processing (DSP) of the Inverse Discrete Fourier Transform (IDFT) at the transmitter and removed after Analogue-to-Digital (ADC) Conversion process at the receiver. The use of cyclic prefix has been a good technique proposed by many authors in mitigating the effect of chromatic dispersion and nonlinearity in fibre-wireless base system. Although, there are some residual dispersion left after chromatic dispersion, these will be mitigated with the use of constellation adjustment method.

However, there is an effect of phase and frequency errors in terms of relationship between the time variation of phase and its effect on the received constellation. However, how these errors are corrected in the digital domain is our concern. First, we consider the case where there is no noise or distortion in the channel and $Qd = 0$, so that there is no I/Q phase imbalance. Now, from the basic theory of OFDM as earlier explained in Sec II, we were made to understand that, OFDM consists of multiple subcarriers that are orthogonal to each other. Henceforth, we deduced that the transmitted baseband OFDM signal is given as;

$$x(k) = \sum_{n\cdot} \sum_{m=0}^{N-1} X_{m,n} b(m-nN_{sym}).\ exp\left(\frac{-j2\pi k_{m\cdot}}{N}\right) \qquad 6$$

where; N is the IFFT size, $X_{m,n}$ is the m-th subcarrier data symbol transmitted on the n-th symbol, N_{sym} is the number of samples in a symbol, and $b(m)$ is the pulse shape of the m-th sample.

We equally deduced that, at the receiver side, the incoming signal $y(t)$ passing through channel can be given as;

$$y(t) = x\,(t) * c(t) + \eta(t) \qquad 7$$

where; $c(t)$ is the impulse response of the channel, [*] is the convolution and $\eta(t)$ is the noise component of the received signal.

Using the formulae for sums and products of angle and with simple manipulation, the time domain received signal samples are given as; Θ_0

$y_m = x_m\, exp\,(j\Theta_m)$ 8

where; Θ_m is the phase error at the receiver for the *mth* sample of the OFDM symbol under consideration. For the case where there is constant phase error, $\Theta_m = \Theta_0$.

Then; $y_m = x_m\, exp\,(j\Theta_0)$ 9

And it is simple to show that;

$y_k = exp\,(j\Theta_m)\, X_k$ 10

The constellation is simply rotated by angle Θ_0 as illustrated in Figure 9 below. In an OFDM system, this would be automatically corrected in the single tap equalizer.

However, let consider the opposite extreme where the phase noise is zero mean and there is no correlation between phase noise samples E{ $\Theta_m\ \Theta_n$ } = 0, m ≠ n, where; E { } denotes the expectation operator. Then taking the FFT gives;

$$y_k = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} y_{m\cdot}\, exp\left(\frac{-j2\pi k_{m\cdot}}{N}\right) \quad 11$$

$$y_k = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x_{m\cdot}\, exp(j\Theta_{m\cdot})exp\left(\frac{-j2\pi k_{m\cdot}}{N}\right) \quad 12$$

If the phase error is small, then applying the small angle approximation $exp(j\Theta_{m\cdot}) \approx 1 + j\Theta_m$.

$$y_k = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x_{m\cdot}\, exp(1 + j\Theta_{m\cdot})exp\left(\frac{-j2\pi k_{m\cdot}}{N}\right) \quad 13$$

$$y_k = X_k + \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x_{m\cdot}\, exp(j\Theta_{m\cdot})exp\left(\frac{-j2\pi k_{m\cdot}}{N}\right) \quad 14$$

$y_k = X_k + N_k$ 15

The demodulated subcarrier y_k is equal to the transmitted subcarrier plus a noise like term, N_k which depends on all of the transmitted subcarriers. The power of N_k can be calculated by using the fact that Θ_m and x_m are statistically independent.

3.5 Constellation Adjustment Method (CAM)

Constellation adjustment method also known as phase rotation is a method of tackling some residual dispersion left after chromatic dispersion and equalization. The steps are outlined below;

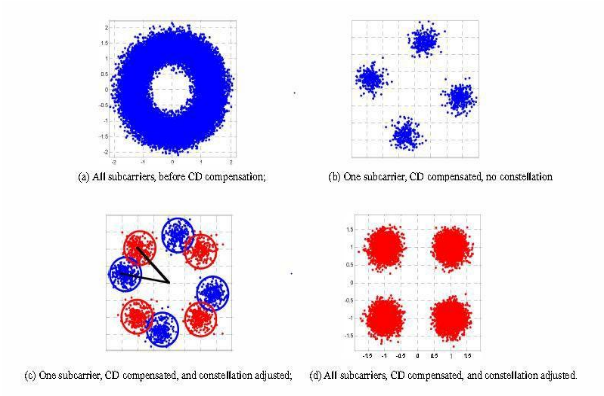
Step 1. Rotate the phase dispersion $\Theta_{(k)}$ for all the subcarriers.

Step 2. Divide the constellation points into quadrants according to the subcarrier modulation format.

Step 3. For each quadrant, calculate the centre of the points, the difference between the phase/amplitude and the theoretical constellation positions.

Step 4. For each quadrant, make adjustment to all subcarriers, to compensate the phase/amplitude discrepancies.

Figure 9 (a) shows one of the Chromatic Dispersion compensated subcarrier, where the phases are compensated by $\Theta_{(k)}$. The residual rotation is still visible in Figure 9 (b). This is compensated as well by the constellation adjustment, as shown in Figure 9 (c). Figure 9 (d) shows the constellation of all compensated subcarriers.


(a) All subcarriers, before CD compensation; (b) One subcarrier, CD compensated, no constellation

(c) One subcarrier, CD compensated, and constellation adjusted; (d) All subcarriers, CD compensated, and constellation adjusted.

Figure 9. Constellation Adjustment for Phase Rotations.

The algorithm for the proposed residual chromatic dispersion compensation is shown in Figure 10 below. It is a smoothing algorithm, that shows sequences in realizing the proposed post compensation dispersion compensation, constellation adjustment method.

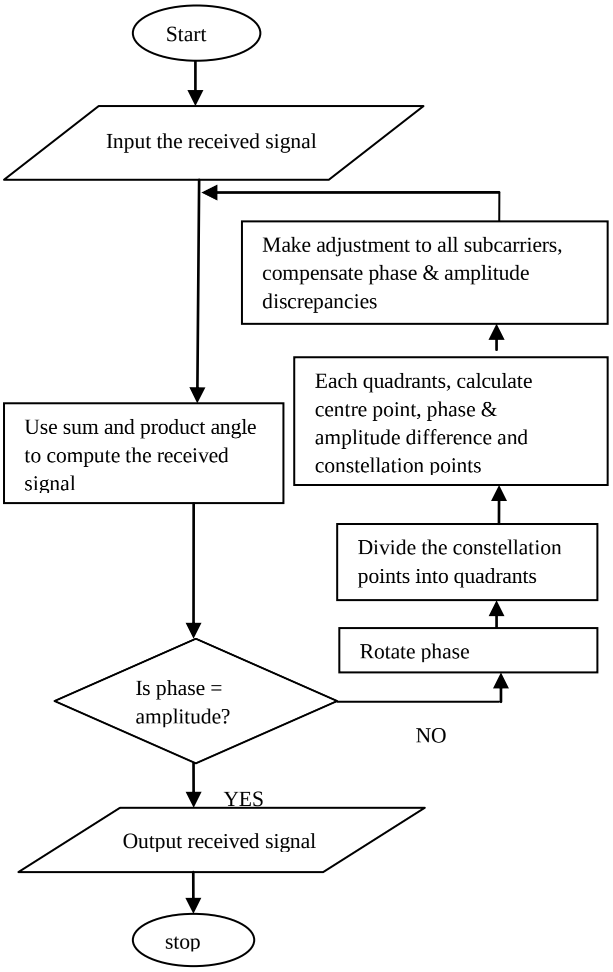


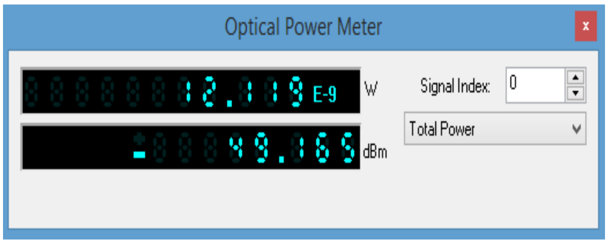
Figure 10. Algorithm for the Compensation of Residual Dispersion

4. RESULTS AND DISCUSSIONS

4.1 Simulation Results for the Proposed System.

The performance evaluation of the proposed system was done using OptiSystem. A single channel of 4-QAM CO-OFDM with a data rate of 107-Gb/s was simulated over various transmission distances (n loops), Optical Signal to Noise Ratio (OSNR) and number of subbands. Note that any change in the fibre physical parameters will distort the form of the received constellation. The OFDM symbol size of 144, which consists of a DFT size of 128 with a guard interval of 16 and a data sample of 82 subcarriers, which consists 5 pilot subcarriers and 41 unused subcarriers were used for the simulation.

Once the simulation is executed, the OptiSystem Analyzer tools will display the optical power by Optical Power Meter, received optical spectrum and RF spectrum by Optical Spectrum and RF Spectrum Analyzers respectively. While the received constellation diagrams are displaced by Electrical Constellation Visualizers and finally, its corresponding Bit Error Rate (BER) by BER Analyzer. Figure 11 shows the received optical power displayed after 4-QAM CO-OFDM Transmitter, while Figure 12 and Figure 13 show the received optical spectrum plotted after fibre link and received RF spectrum plotted after radio link respectively. Figure 14 and Figure 15 show the received 4-QAM CO-OFDM constellation before CD and after CD respectively. Finally, Figure 16 shows the BER value displayed by the BER Eye Diagram.



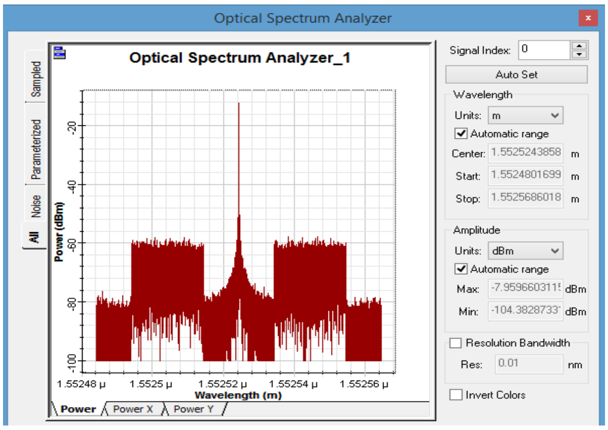
12.119 E-9 W

-49.165 dBm

Signal Index: 0

Total Power

Figure 11. Received Optical Power after 4-QAM CO-OFDM Transmitter.



Signal Index: 0

Wavelength

Units: m

☑ Automatic range

Center: 1.5525243858 m

Start: 1.5524801699 m

Stop: 1.5525686018 m

Amplitude

Units: dBm

☑ Automatic range

Max: -7.959660311 dBm

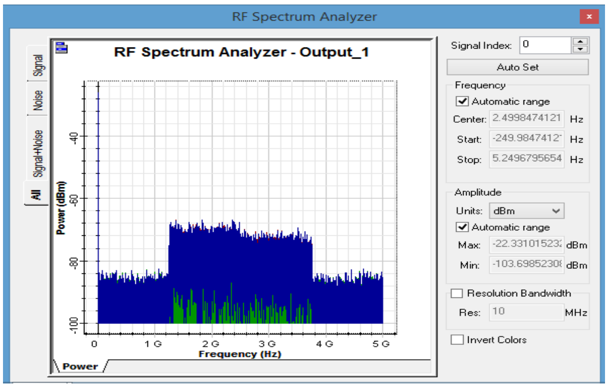
Min: -104.3828733 dBm

☐ Resolution Bandwidth

Res: 0.01 nm

☐ Invert Colors

Figure 12. Received Optical Spectrum after Fibre Link



Signal Index: 0

Frequency

☑ Automatic range

Center: 2.4998474121 Hz

Start: -249.9847412 Hz

Stop: 5.2496795654 Hz

Amplitude

Units: dBm

☑ Automatic range

Max: -22.33101523 dBm

Min: -103.6985230 dBm

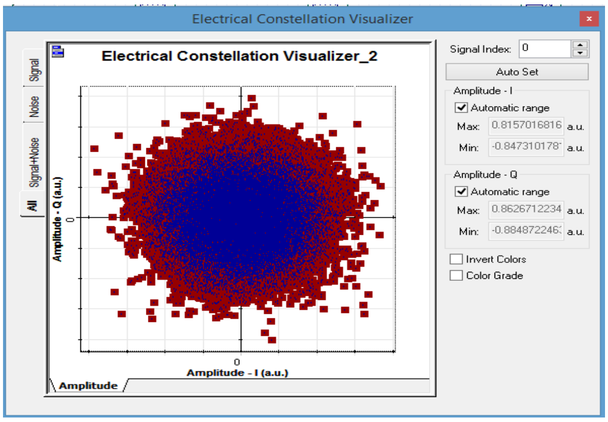
☐ Resolution Bandwidth

Res: 10 MHz

☐ Invert Colors

Figure 13. Received Radio Frequency Spectrum after Radio Link

The 4-QAM COFDM constellation shown in Figure 14 resembled a doughnut shape, this is due to non addition of cyclic prefix in the transmission. Consequently, the inter symbol interference, inter carrier interference and other impairments associated with fiber lead to poor quality signal. However, with addition of cyclic prefix, Figure 15 is obtained after equalization and employing the proposed technique; combination of CO-OFDM technique with Constellation Adjustment Method post compensated. A far better signal quality is realized as compared to Non-Return-Zero system.



Signal Index: 0

Amplitude - I

☑ Automatic range

Max: 0.8157016816 a.u.

Min: -0.847310178 a.u.

Amplitude - Q

☑ Automatic range

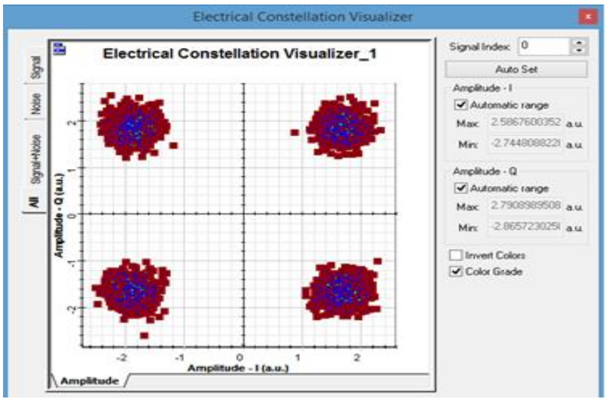
Max: 0.8626712234 a.u.

Min: -0.884872246 a.u.

☐ Invert Colors

☐ Color Grade

Figure 14. Received Optical Constellations for 4-QAM CO-OFDM before CD.



Signal Index: 0

Amplitude - I

☑ Automatic range

Max: 2.5867600352 a.u.

Min: -2.744808822 a.u.

Amplitude - Q

☑ Automatic range

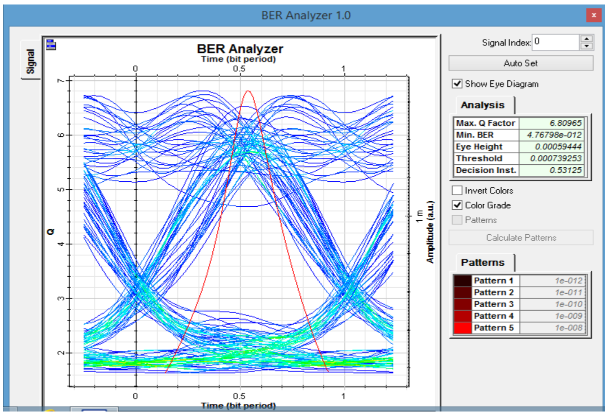
Max: 2.7908989508 a.u.

Min: -2.865723025 a.u.

☐ Invert Colors

☑ Color Grade

Figure 15. Received Optical Constellations for 4-QAM CO-OFDM after CD.



Signal Index: 0

☑ Show Eye Diagram

Analysis

Max. Q Factor	6.80965
Min. BER	4.76798e-012
Eye Height	0.00059444
Threshold	0.000739253
Decision Inst.	0.53125

☐ Invert Colors
☑ Color Grade
☐ Patterns

Patterns

Pattern 1	1e-012
Pattern 2	1e-011
Pattern 3	1e-010
Pattern 4	1e-009
Pattern 5	1e-008

Figure 16. BER Eye Diagram for 4-QAM CO-OFDM Post Compensation.

The BERs of NRZ, 4 QAM-OFDM, 4 PSK-OFDM systems against the same OSNR was estimated. The proposed 4-QAM CO-OFDM and 4-PSK CO-OFDM systems use a 10 GHz brick-wall optical filter before the coherent receivers, while the NRZ system uses a 20 GHz brick-wall optical filter. The NRZ transmitter and receiver have a zero linewidth laser and a 7.5 GHz fourth order Bessel filter respectively.

Figure 17 shows BER v/s OSNR, calculated over a bandwidth of 12.5-GHz. NRZ needs a 1-dB and 0.5-dB better OSNR than 4-QAM CO-OFDM and 4-PSK CO-OFDM for Min. BER of 4.76798e-12 and Max. Q factor of 6.80965 respectively. Hence, the proposed CO-OFDM chromatic dispersion post compensated systems outperformed or rather showed superior BER in comparison to the conventional NRZ system in the case of the same number of parameters.

In addition, phase rotation of the proposed system removed the discontinuity and compensated for the residual dispersion left after chromatic dispersion compensation. Hence, adoption of the smoothing algorithm to the estimated channel improved BER performance in the proposed system.

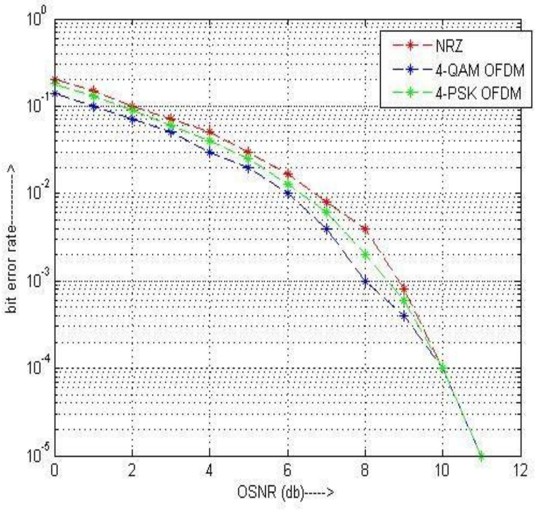


Figure 17. BER v/s OSNR for OFDM and NRZ systems.

4.2 Simulation Results at Different Loops (Distances)

The statistical analysis and results of the Min. BER and Max. Q factor for different distances after incorporating a dispersion management scheme into the coherent OFDM simulator is shown in Table 6 below. Note; in each of the loops (multiple of 100km), the simulation is done for 10 iterations and the Min. BER and Max. Q factors were recorded.

Table 6. Results of Min. BER and Max. Q Factors for Different Distances.

S/No	Distance (km)	Min. BER	Max. Q factor
1	100	4.76798e-12	6.80965
2	200	6.19523e-12	6.77137
3	300	6.30997e-12	6.76898
4	400	6.98688e-12	6.75381
5	500	7.11726e-12	6.75097
6	600	7.76709e-12	673832
7	700	8.38695e-12	6.72724
8	800	9.17197e-12	6.71401
9	900	9.40691e-12	6.71085
10	1000	1.02198e-11	6.69855

From the simulation result shown in Table 6, we can deduced that a better signal quality is gotten at a closer distance but degrades as the distance increases. This is due to transmission impairments (fading, chromatic dispersion, etc) associated with long-haul transmission network. A combination of addition of Cyclic Prefix (CP) and Constellation Adjustment Method (CAM) compensated for these impairments.

5. CONCLUSION

All the transmission impairments including higher order dispersion were compensated. To effectively and efficiently compensate for the residual dispersion, CO-OFDM combined with the Constellation Adjustment Method was proposed. The proposed implementation required no computational complexity or rather extra digital signal processing to implement. The non high frequency requirement for its operation is another added advantage to the proposed system. The simulation results showed that the proposed system theoretically achieved transmission speed of 107-Gb/s single channel CO-OFDM over 1000km optical transmission fibre.

6. ACKNOWLEDGMENTS

Our immeasurable thanks to the experts who have contributed towards development of the Optisystem Software which aided us in our simulation. More so, to the Sony Ericson Research Center and Communication Lab, Modibbo Adama University of Technology, Yola, Adamawa State, Nigeria, which provided us with the physical fiber layers for this research.

RFERENCES

[1] R. W. Chang, "Synthesis of band-limited orthogonal signals for multichannel data transmission," *Bell Systems Tech. Journal,* vol. vol. 45, pp. pp.1775–1796, 1966.

[2] S. John, E. Akinola, F. Ibikunle, C. Ndujiuba, and B. Akinaade, "Modeling of Orthogonal Frequency Division Multiplexing (OFDM) for Transmission in Broadband Wireless Communications," *Journal of Emerging Trends in Computing and Information Sciences,* vol. 3, pp. 534-539, 2012.

[3] L. J. Cimini Jr, "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *Communications, IEEE Transactions on,* vol. 33, pp. 665-675, 1985.

[4] S. B. Alexander, M. Y. Frankel, S. W. Chaddick, R. C. Litz, and C. D. Smith, "High-speed optical transponder systems," ed: Google Patents, 2015.

[5] S. Amiralizadeh, A. T. Nguyen, C. S. Park, and L. A. Rusch, "Single-Fiber Lightwave Centralized WDM-OFDMA-PON With Colorless Optical Network Units," *Journal of Optical Communications and Networking,* vol. 8, pp. 196-205, 2016.

[6] M. Chen, X. Xiao, Z. R. Huang, J. Yu, F. Li, Q. Chen, *et al.*, "Experimental demonstration of an IFFT/FFT size efficient DFT-spread OFDM for short reach optical transmission systems," *Journal of Lightwave Technology,* vol. 34, pp. 2100-2105, 2016.

[7] N. Chide, S. Deshmukh, and P. Borole, "Implementation of OFDM System using IFFT and FFT," *International Journal of Engineering Research and Applications (IJERA),* vol. 3, pp. 2009-2014, 2013.

[8] N. Kaneda, T. Pfau, H. Zhang, J. Lee, Y.-K. Chen, C. J. Youn, *et al.*, "Field demonstration of 100-Gb/s real-time coherent optical OFDM detection," *Journal of Lightwave Technology,* vol. 33, pp. 1365-1372, 2015.

[9] E. Giacoumidis, S. Mhatli, T. Nguyen, S. T. Le, I. Aldaya, M. E. McCarthy, *et al.*, "Comparison of DSP-based nonlinear equalizers for intra-channel nonlinearity compensation in coherent optical OFDM," *Optics letters,* vol. 41, pp. 2509-2512, 2016.

[10] B. R. Salzberg, "Performance of an efficient parallel data transmission system," *Transmission Communication Technology Journal of IEEE* vol. 15, pp. 805-813, Dec, 1967.

[11] a. S. B. W. J. Salz, "Fourier transform communication system " *In Proceedings of the first ACM symposium on Problems in the optimization of data communications systems, ACM,* pp. pp. 99-128, October 1969.

[12] A. Peled and A. Ruiz, "Frequency domain data transmission using reduced computational complexity algorithms," in *Acoustics, Speech, and Signal Processing, IEEE International Conference on ICASSP'80.*, 1980, pp. 964-967.

[13] R. Lassalle and M. Alard, "Principles of modulation and channel coding for digital broadcasting for mobile receivers," *EBU Tech. Rev,* vol. 224, pp. 168-190, 1987.

[14] W. Shieh, X. Yi, and Y. Tang, "Transmission experiment of multi-gigabit coherent optical OFDM systems over 1000 km SSMF fibre," *Electronics letters,* vol. 43, p. 1, 2007.

[15] A. Sano, E. Yamada, H. Masuda, E. Yamazaki, T. Kobayashi, E. Yoshida, *et al.*, "No-guard-interval coherent optical OFDM for 100-Gb/s long-haul WDM transmission," *Journal of Lightwave Technology,* vol. 27, pp. 3705-3713, 2009.

[16] Y. Ma, Q. Yang, Y. Tang, S. Chen, and W. Shieh, "1-Tb/s single-channel coherent optical OFDM transmission with orthogonal-band multiplexing and subwavelength bandwidth access," *Lightwave Technology, Journal of,* vol. 28, pp. 308-315, 2010.

[17] D. Patel, V. K. Singh, and U. Dalal, "Assessment of fiber chromatic dispersion based on elimination of second-order harmonics in optical OFDM single sideband modulation using Mach Zehnder Modulator," *Fiber and Integrated Optics,* vol. 35, pp. 181-195, 2016.

[18] Q. Yang, Z. He, Z. Yang, S. Yu, X. Yi, and W. Shieh, "Coherent optical DFT-spread OFDM transmission using orthogonal band multiplexing," *Optics express,* vol. 20, pp. 2379-2385, 2012.